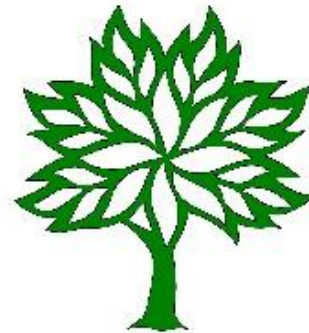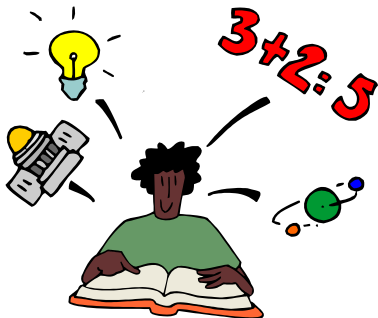# Written Calculation Progression
Key Stages 1 - 2

Darell School

Created: September 2015

Darell School
Pencil and paper procedures

Background to the progression

This progression contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement.

Although the focus of the progression is on pencil and paper procedures, it is important to recognise that the ability to calculate mentally and to recall apply knowledge rapidly and accurately is a crucial element of the new mathematics curriculum. The mental methods in the 2014 *National Curriculum for Mathematics* will be taught systematically from Reception onwards and pupils will be given regular opportunities to develop the necessary skills; for example, through the "Maths Mountain" Strategy and through the use of *Numicon* in the Foundation Stage and Years 1 and 2. Mental calculation, however, is not at the exclusion of written recording and should be seen as complementary to and not as separate from it.   In every written method, there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas; therefore, written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

During their time at Darell School, children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:
- developing the use of pictures and a mixture of words and symbols to represent numerical activities;
- using standard symbols and conventions;
- use of jottings to aid a mental strategy and
- use of pencil and paper procedures.

This progression concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Children will, therefore, always be encouraged to look at a calculation/problem and then decide which is the best method to choose – pictures, mental calculation with or without jottings, or structured recording. Our long-term aim is for children to be able to select an efficient method of their choice (whether this be mental or written) that is appropriate for a given task. They will do this by always asking themselves:

'Can I do this in my head?'
'Can I do this in my head using drawings or jottings?'
'Do I need to use a pencil and paper procedure?'

It is vital that stages of calculation are not rushed through. Many adults find mathematics challenging because they were taught to use complex methods before they were ready. It is important, therefore, that we give children time to gain a real understanding of the mathematics they are using before they move on to more abstract methods of recording. This may mean that a child in Year 4 will still be using strategies shown as appropriate for Year 2; however, for some children it is only through constant practice of one method that true understanding comes. Other children may benefit from moving quickly through the stages or missing stages out all together. It is in this way that we ensure all children have the opportunity to experience an individualised curriculum.

Addition

Year 1	Year 2	Year 3

+ = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

2 = 1+ 1
2 + 3 = 4 + 1
3 = 3
2 + 2 + 2 = 4 + 2

Missing numbers need to be placed in all possible places.

3 + 4 = □   □ = 3 + 4
3 + □ = 7   7 = □ + 4
□ + 4 = 7   7 = 3 + □
□ + ∇ = 7   7 = □ + ∇

Activities
Children should have access to a wide range of counting equipment, everyday objects, as well as hoops, sorting trays, number tracks and numbered number lines.

Teacher modelling
Drawing jumps on numbered number lines and hundred squares to support understanding of the mental method

Children
To create their own jumps using rulers, fingers, pens, bodies, etc.
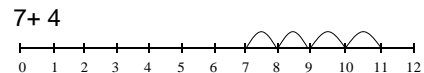
7+ 4

0 1 2 3 4 5 6 7 8 9 10 11 12

+ = signs and missing numbers
Continue using a range of equations as in Year 1, but with appropriate, larger numbers.
Extend to
14 + 5 = 10 + □
and
32 + □ + □ = 100   35 = 1 + □ + 5

Partition into tens and ones and recombine
       12 + 23 =
10 + 2   20 + 3
    10 + 20 = 30
      2 + 3 = 5
30 + 5 = 35

Count on in tens and ones
Use of a numbered hundred square should be emphasised before moving on to an empty number line.

23 + 12 =

          +10   +2
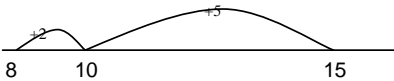
23   33   35
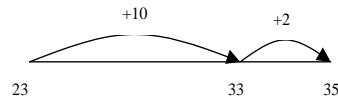
Partitioning and bridging through 10.
The steps in addition often bridge through a multiple of 10

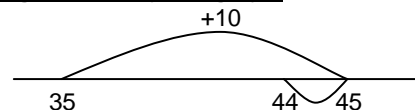Children should be able to partition the 7 to relate adding the 2 and then the 5.

8 + 7 = 15

          +2   +5

8   10   15

Add 9 or 11 by adding 10 and adjusting by 1
Use of a numbered hundred square should be emphasised before moving on to an empty number line.

Add 9 by adding 10 and adjusting by 1
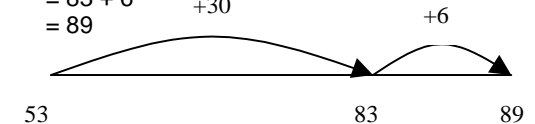35 + 9 = 44   +10

35   44 45

+ = signs and missing numbers
Continue using a range of equations as in Years 1-2, but with appropriate, larger numbers.

Partition into tens and ones
- Partition both numbers and recombine.
- Count on by partitioning the second number only
36 + 53 = 53 + 30 + 6
        = 83 + 6
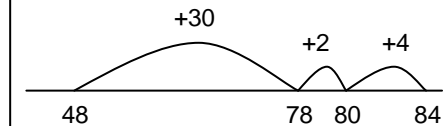   = 89   +30   +6

53   83   89

Add a near multiple of 10 to a two-digit number
Secure mental methods by using a number line to model the method. Continue as in Year 2 but with appropriate numbers
35 + 19 is the same as 35 + 20 – 1.

Children need to be secure adding multiples of 10 to any two-digit number including those that are not multiples of 10.
48 + 36 = 84

              +30
                      +2   +4

48   78 80   84

Pencil and paper procedures
83 + 42 = 125

1. Horizontal expansion *then* **2. Vertical expansion**
      80 + 3   83
+   40 + 2   + 42
    120 + 5 = 125   5
                                  120
                                  125

Addition

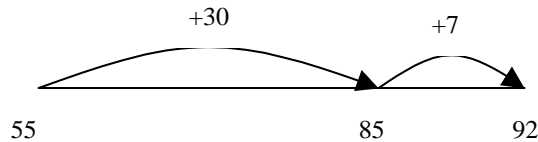| Year 4	Year 5	Year 6

+ = signs and missing numbers
Continue using a range of equations as in Years 1-3, but with appropriate numbers.

Partition into tens and ones and recombine
Either partition both numbers and recombine or partition the second number only
55 + 37 = 55 + 30 + 7
      = 85 + 7
      = 92

+30   +7

55   85   92

Add the nearest multiple of 10, then adjust
Continue as in Years 2-3, but with appropriate numbers
63 + 29 is the same as 63 + 30 - 1

Pencil and paper procedures
367 + 185 = 431

First   **then**

  300 + 60 + 7   367
+ 100 + 80 + 5   + 185
  400 + 140 + 12   12
                          140
                          400
                          552

leading to

  367
 +185
  552
   1 1

Recording of 'carrying' can be done at the bottom or at the top.

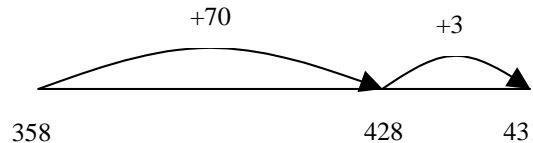Extend to decimals in the context of money.

+ = signs and missing numbers
Continue using a range of equations as in Years 1-4, but with appropriate numbers.

Partition into hundreds, tens and ones and recombine
Partition the second number only and recombine

358 + 73 = 358 + 70 + 3
        = 428 + 3
        = 431

+70   +3

358   428   431

Add or subtract the nearest multiple of 10 or 100, then adjust
Continue as in Years 2- 4, but with appropriate numbers
458 + 79 = is the same as 458 + 80 - 1

Pencil and paper procedures
Extend to numbers with at least four digits
3587 + 675 = 4262

  3587
+ 675
  4262
  1 1 1

Revert to expanded methods if the children experience any difficulty.
Adding more than two numbers together e.g. 3891 + 245 + 37
Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).
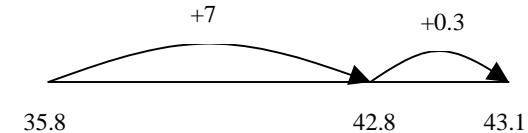   72.8
+ 54.6
   127.4

+ = signs and missing numbers
Continue using a range of equations as in Years 1-5, but with appropriate numbers.

Partition into hundreds, tens, ones and decimal fractions and recombine
Either partition both numbers and recombine or partition the second number only e.g.
35.8 + 7.3 = 35.8 + 7 + 0.3
          = 42.8 + 0.3
          = 43.1

+7   +0.3

35.8   42.8   43.1

Add the nearest multiple of 10, 100 or 1000, then adjust
Continue as in Years 2-5, but with appropriate numbers, including extending to adding 0.9, 1.9, 2.9, etc.

Pencil and paper procedures
Extend to numbers with any number of digits and decimals with 1, 2 and/or 3 decimal places.
13.86 + 9.481 = 23.341

  13.860
+ 9.481
  23.341
   1 1 1

Zero added into the thousandths column to ensure the same number of decimal places.

Revert to expanded methods if the children experience any difficulty.

Children to add more than two numbers together (with different numbers of digits and including decimal numbers) e.g. 368.4 + 98.27 + 0.08

Subtraction

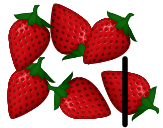| Year 1 | Year 2 | Year 3 |

- = signs and missing numbers

7 - 3 = ☐   ☐ = 7 - 3
7 - ☐ = 4   4 = ☐ - 3
☐ - 3 = 4   4 = 7 - ☐
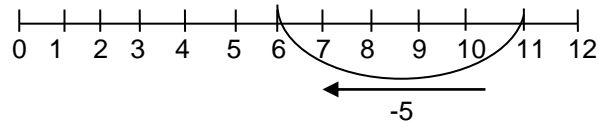☐ - ∇ = 4   4 = ☐ - ∇

- Understand subtraction as 'take away'

- Use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number.

I have 11 toy cars. There are 5 cars too many to fit in the garage. How many cars fit in the garage?

-5

- Find a 'difference' by counting up;

I have saved 5p. The socks that I want to buy cost 11p. How much more do I need in order to buy the socks?

+6

0 1 2 3 4 5 6 7 8 9 10 11 12
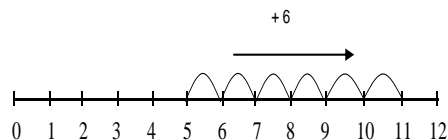
Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences

Recording by
 - drawing jumps on prepared lines and hundred squares
 - constructing own lines

- = signs and missing numbers
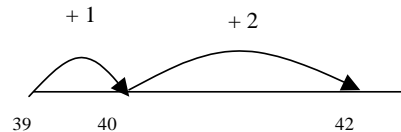Continue using a range of equations as in Year 1, but with appropriate numbers.
Extend to 14 + 5 = 20 - ☐
☐ - 5 = 13

Find a small difference by counting up
Use of a numbered number line or hundred square should be emphasised before moving on to an empty number line.
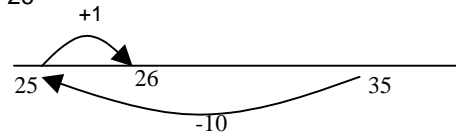42 – 39 = 3

+ 1   + 2

39   40   42

Subtract 9 or 11. Begin to add/subtract 19 or 21
Use of a numbered hundred square should be emphasised before moving on to an empty number line.
35 – 9 = 26

+1

25   26   35
          -10
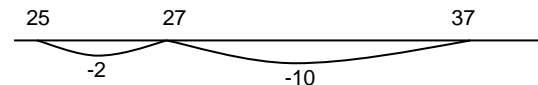
Use known number facts and place value to subtract
Use of a numbered hundred square should be emphasised before moving on to an empty number line.
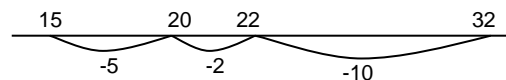
(partition second number only)
37 – 12 = 37 – 10 – 2
        = 27 – 2
        = 25

25   27   37
    -2   -10

Bridge through 10 where necessary
32 – 17 = 15

15   20   22   32
    -5   -2   -10

- = signs and missing numbers
Continue using a range of equations as in Years 1-2, but with appropriate numbers

Find a small difference by counting up
Use of a numbered number line or hundred square should be emphasised before moving on to an empty number line.
Continue as in Years 1-2, but with appropriate numbers
102 – 97 = 5

Subtract mentally a 'near multiple of 10' from a two-digit number
Continue as in Year 2, but with appropriate numbers
78 – 49 is the same as 78 – 50 + 1

Use known number facts and place value to subtract
Use of a numbered hundred square should be emphasised before moving on to an empty number line.
Continue as in Year 2, but with appropriate numbers, e.g.
97 – 15 = 72

82   87   977
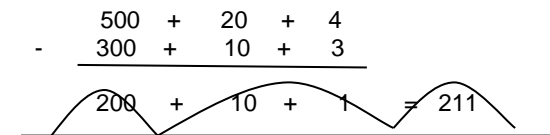
        -5
              -10
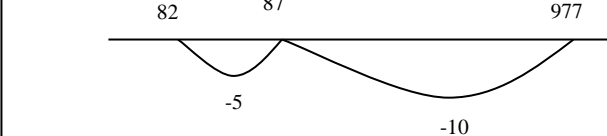
With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient, such as 57 – 12, 86 – 77 or 43 – 28.

Pencil and paper procedures

Partitioning with no exchange.
524 – 313 = 211

```
      500  +  20  +  4
  -   300  +  10  +  3
      200  +  10  +  1  =  211
```

Subtraction

Year 4	Year 5	Year 6

- = signs and missing numbers
Continue using a range of equations as in Years 1-3, but with appropriate numbers.

Find a small difference by counting up
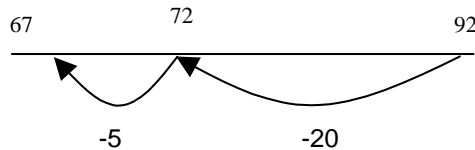5003 – 4996 = 7
This can be modelled on an empty number line (see complementary addition below). Children should be encouraged to use known number facts to reduce the number of steps.

Subtract the nearest multiple of 10, then adjust.
Continue as in Years 2-3, but with appropriate numbers.

Use known number facts and place value to subtract
92 – 25 = 67

-5   -20

Pencil and paper procedures
Complementary addition

For those children with a secure mental image of the number line they could record the jumps only:

754 – 86 = 668

   14 (100)
  600 (700)
   54 (754)
  668

Partitioning with exchange

Progression of decomposition is shown in the appendix.

- = signs and missing numbers
Continue using a range of equations as in Years 1-4, but with appropriate numbers.

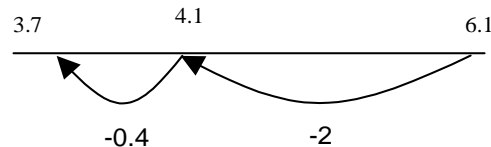**Find a difference by counting up**
8006 – 2993 = 5013
This can be modelled on an empty number line (see complementary addition below).

Subtract the nearest multiple of 10 or 100, then adjust.
Continue as in Year 2-4, but with appropriate numbers.

Use known number facts and place value to subtract
6.1 – 2.4 = 3.7

3.7   4.1   6.1

-0.4   -2

Pencil and paper procedures
Complementary addition
754 – 286 = 468

  14 (300)   can be refined to   14 (300)
400 (700)   454 (754)
  54 (754)   468
468

Reduce the number of steps to make the calculation more efficient.

Transition from the partitioning method used in Year 4 (see appendix) to using the compact method of decomposition, including decimals.
78.4 – 25.6 = 42.8

$$7\overset{7}{\cancel{8}}.\overset{1}{4}$$
$$-\ 2\ 5\ .\ 6$$
$$\overline{5\ 2\ .\ 8}$$

- = signs and missing numbers
Continue using a range of equations as in Years 1-5, but with appropriate numbers.
Find a difference by counting up
8000 – 2785 = 5215
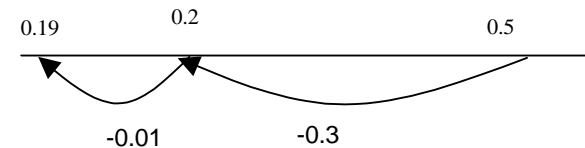To make this method more efficient, the number of steps should be reduced to a minimum through children knowing
- Complements to 1, involving decimals to two decimal places (0.16 + 0.84)
- Complements to 10, 100 and 100

Subtract the nearest multiple of 10, 100 or 1000, then adjust
Continue as in Years 2-5, but with appropriate numbers.
Use known number facts and place value to subtract

0.5 – 0.31 = 0.19

0.19   0.2   0.5

-0.01   -0.3

Pencil and paper procedures
Complementary addition: 6467 – 2684 = 3783

6467 – 2684 = 3783

  16 (2700)   can be refined to   316 (3000)
300 (3000)   3467 (6467)
 3467 (6467)   3783
3783

Reduce the number of steps to make the calculation more efficient.

Extend to 2 places of decimals and decomposition.
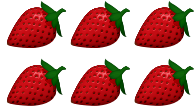Progression of decomposition is shown in the appendix.

Multiplication

Year 1	Year 2	Year 3

Year 1

Multiplication is related to doubling and counting groups of the same size.

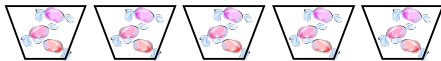| Looking at columns	Looking at rows
2 + 2 + 2	3 + 3
3 groups of 2	2 groups of 3

Counting using a variety of practical resources

Counting in 2s, e.g., counting socks, shoes, animals' legs…
Counting in 5s, e.g., counting fingers, fingers in gloves, toes…
Counting in 10s, e.g., fingers, toes…

Pictures/marks

There are 3 sweets in one bag.
How many sweets are there in 5 bags?
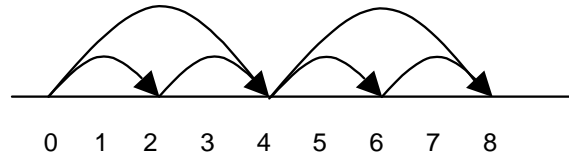
Year 2

<u>x = signs and missing numbers</u>

7 x 2 = ☐	☐ = 2 x 7
7 x ☐ = 14	14 = ☐ x 7
☐ x 2 = 14	14 = 2 x ☐
☐ x ∇ = 14	14 = ☐ x ∇

<u>Arrays and repeated addition</u>

• • • •   4 x 2 or 2 + 2 + 2 + 2
• • • •
2 x 4 or 4 + 4

0 1 2 3 4 5 6 7 8

<u>Doubling multiples of 5 up to 50</u>

2 x 15 = 30

Partition

Children need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways: 6 = 5 + 1 so for example, double 6 is the same as double five add double one.

10 + 5

AND double 15

20 + 10 = 30

OR

x	10	5
2	20	10	= 30

(smaller number along left side)

Year 3

<u>x = signs and missing numbers</u>

Continue using a range of equations as in Year 2 but with appropriate numbers.

<u>Arrays and repeated addition</u>

Continue to understand multiplication as repeated addition and continue to use arrays (as in Year 2).

<u>Doubling multiples of 5 up to 50</u>

35 x 2 = 70

Partition   (smaller number along

X	30	5	left side)
2	60	10	=70

<u>Use known facts and place value to carry out simple</u>

<u>multiplications</u>

Use the same method as above (partitioning)

72 x 38 = 2100 + 560 + 60 + 16 = 2736

x	70	2
30	2100	60
8	560	16

Multiplication

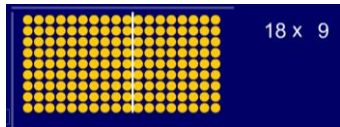| Year 4	Year 5	Year 6

Year 4

x = signs and missing numbers
Continue using a range of equations as in Years 2 and 3, but with appropriate numbers

Partition
Continue to use arrays:

18 x 9

18 x 9 = 162

18 x 9 = (10 x 9) + (8 x 9) = 162

OR

Use the grid method of multiplication (as below)
(smaller number along left side)

Pencil and paper procedures

Continue use of grid method, as taught in Year 3.

Move on to using the expanded method of multiplication

```
234 x 8          234
               x   8
                  32   (4 x 8)
                 240   (30 x 8)
                1600   (200 x 8)
                1872
```

Year 5

x = signs and missing numbers
Continue using a range of equations as in Years 2-4 but with appropriate numbers

Partition
47 x 6 = 282

47 x 6 = (40 x 6) + (7 x 6) = 282

Pencil and paper procedures

Use grid method to introduce multiplying decimals

3.7 x 4 = 14.8

X	3	0.7
4	12	2.8

= 14.8

Use expanded method of multiplication (see Year 4) then move on to the short method of multiplication.

45 x 38 =

```
     4 5
  X  3 8
    ─────
     4 0    (5 x 8)
   3 2 0    (40 x 8)
   1 5 0    (5 x 30)
 1 2 0 0    (40 x 30)
 ─────────
 1 7 1 0
 ─────────
   1
```

```
      1 4
     4 5
   x 3 8
   ─────
   3 6 0
 1 3 5 0
 ─────────
 1 7 1 0
 ─────────
   1
```

Year 6

x = signs and missing numbers
Continue using a range of equations as in Years 2-5, but with appropriate numbers

Partition
87 x 6 = 522

87 x 6 = (80 x 6) + (7 x 6) = 522

OR

Use the short method of multiplication (see below)

Pencil and paper procedures

```
              1
231 x 16     231
           x  16
           1386
           2310

           ─────
           3696
```

Use the short method to multiply calculations involving decimals

14.6 x 7 =

First remove the decimal point

```
     3 4
    1 4 6
  X     7
  ───────
  1 0 2 2
```

Then reintroduce the decimal into the answer = 102.2

Division

| **Year 1** | **Year 2** | **Year 3** |

Year 1

Sharing

Requires secure counting skills
-see counting and understanding number strand
Develops importance of one-to-one correspondence

Sharing – 6 sweets are shared between 2 people. How many do they have each?

Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops, etc.

Grouping
Sorting objects into 2s/3s/4s, etc.
How many pairs of socks are there?

There are 12 crocus bulbs. Plant 3 in each pot. How many pots are there?
Jo has 12 Lego wheels. How many cars can she make?

Year 2

÷ = signs and missing numbers

$6 \div 2 = \square$ $\square = 6 \div 2$
$6 \div \square = 3$ $3 = 6 \div \square$
$\square \div 2 = 3$ $3 = \square \div 2$
$\square \div \nabla = 3$ $3 = \square \div \nabla$
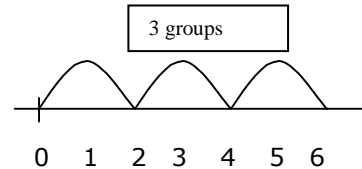
Grouping
Link to counting and understanding number strand
Count up to 100 objects by grouping them and counting in tens, fives or twos;
Find one half, one quarter and three quarters of shapes and sets of objects
$6 \div 2$ can be modelled as:
There are 6 strawberries. How many 2s in 6?
How many people can have 2 each?

$6 \div 2$ can be modelled as:

3 groups

0 1 2 3 4 5 6

In the context of money count forwards and backwards using 2p, 5p and 10p coins

Practical grouping, e.g., in PE

12 children get into teams of 4 to play a game. How many teams are there?

Sharing
Continue as in Year 1, using a more formal recording method with the division symbol.

Link division facts to times tables.
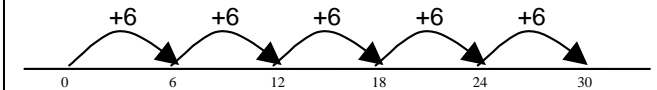
Year 3

÷ = signs and missing numbers
Continue using a range of equations as in Year 2 but with appropriate numbers.

Understand division as sharing and grouping
Sharing and grouping
$30 \div 6$ can be modelled as:
grouping – groups of 6 placed on no. line and the number of groups counted

+6   +6   +6   +6   +6

0   6   12   18   24   30

Sharing – sharing among 6, the number given to each person

Remainders
$16 \div 3 = 5 \text{ r}1$
Grouping – How many 3s in 16; how many left over?
Sharing – 16 shared between 3; how many left over?

0   3   6   9   12   15 16

Pencil and paper procedures

$72 \div 5$ lies between $50 \div 5 = 10$ and $100 \div 5 = 20$
* Partition the dividend into multiples of the divisor:
    $72 = 50 + 22$
        $50 \div 5 = 10$
        $22 \div 5 = 4 \text{ r } 2 \rightarrow 10 + 4 \text{ r } 2 = 14 \text{ r } 2$

72

50 + 22

10 + 4 r 2

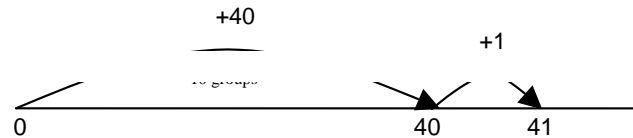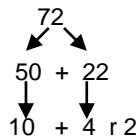Answer : 14 r 2

Division

Year 4	Year 5	Year 6

÷ = signs and missing numbers
Continue using a range of equations as in Years 2-3, but with appropriate numbers.

Remainders
$41 ÷ 4 = 10$ r1

+40

+1

0   40   41

$41 = (10 \times 4) + 1$

Pencil and paper procedures
$72 ÷ 5$ lies between $50 ÷ 5 = 10$ and $100 ÷ 5 = 20$
* Partition the dividend into multiples of the divisor:
$72 = 50 + 22$
        $50 ÷ 5 = 10$
        $22 ÷ 5 = 4r2 \rightarrow 10 + 4r 2 = 14$ r 2

72
50 + 22
10 + 4 r 2

Answer :   14 r 2

Then move on to the expanded bus stop method for division

$$10 + 4 \ \ r 2 \ = \ 14 \ r 2$$

```
5 | 7 2
  -  5 0
  ───────
     2 2
  -  2 0
  ───────
       2
```

÷ = signs and missing numbers
Continue using a range of equations as in Years 2-4, but with appropriate numbers.

Sharing and grouping
Continue to understand division as both sharing and grouping (repeated subtraction).

Remainders
Quotients expressed as fractions or decimals
$61 ÷ 4 = 15 ¼$ or 15.25

Pencil and paper procedures
$196 ÷ 6$ lies between $180 ÷ 6 = 30$ and $240 ÷ 6 = 40$

Expanded bus stop method

```
     30 + 2 r 4
 6 | 196
 -   180          Answer: 32 r 4
     ───
      16
 -    12
     ───
       4
```

Long division

```
      3 2  r 4
 6 | 1 9 6
 -   1 8
     ────
       16
 -     12
      ───
        4
```

÷ = signs and missing numbers
Continue using a range of equations as in Years 2-5, but with appropriate numbers.

Sharing and grouping
Continue to understand division as both sharing and grouping (repeated subtraction).

Remainders

Quotients expressed as fractions or decimals
$676 ÷ 8 = 84 ½$ or 84.5

Pencil and paper procedures
$977 ÷ 36$ is approximately $1000 ÷ 40 = 25$

Compact bus stop method

```
      0  3  2  r 4
 6 | 4 ₁9 ₁6
```

$46 . 8 ÷ 3 = 15.9$

```
      1  5 . 9
 3 | 4 ₁7 . ₂7
```

Continue to use long division (see Year 5)